Topic 7

Algebraic Methods

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

PEARSON

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 26

Q1

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when f(x) is divided by (x −1)

(2)

(b) Use the factor theorem to show that (x+1) is a factor of f(x)

(2)

(c) Factorise f(x) completely.

(4)

(Total for Question 1 is 9 marks)

Q2

(a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$

(ii) $x + 2$

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0$$

(4)

(Total for Question 2 is 7 marks)

Q3

$$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7

(a) Show that $a + b = 3$

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8

(b) Find the value of a and the value of b

(5)

(Total for Question 3 is 7 marks)

Q4

Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(Total for Question 4 is 4 marks)

End of Questions

Bronze Mark Scheme

Q1

Question Number	Scheme		Marks
(a)	$f(x) = 2x^3 - 7x^2 - 5x + 4$ Remainder $= f(1) = 2 - 7 - 5 + 4 = -6$ $= -6$	Attempts $f(1)$ or $f(-1)$. -6	M1 A1 [2]
(b)	$f(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ and so $(x+1)$ is a factor.	Attempts $f(-1)$. $f(-1) = 0$ with no sign or substitution errors **and for conclusion.**	M1 A1 [2]
(c)	$f(x) = \{(x+1)\}(2x^2 - 9x + 4)$ $= (x+1)(2x-1)(x-4)$ (**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*).		M1 A1 dM1 A1 [4] 8

(a)	M1 for ***attempting*** either $f(1)$ or $f(-1)$. Can be implied. Only one slip permitted. M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of x. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6. Award M1A1 for -6 without any working.
(b)	M1: attempting only $f(-1)$. A1: must correctly show $f(-1) = 0$ and give a conclusion ***in part (b) only***. **Note**: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a preamble, eg: "If $f(-1) = 0$, $(x+1)$ is a factor…." **Note: Long division scores no marks in part (b). The factor theorem is required.**
(c)	1st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." $(2x^2 \pm ax \pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $f(x)$ by $(x-1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a). 1st A1: For seeing $(2x^2 - 9x + 4)$. 2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly. 2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}$, $x = \frac{1}{2}$, 4, and not list all three factors. Award these responses M1A1M1A0. **Alternative:** 1st M1: For finding either $f(4) = 0$ or $f\left(\frac{1}{2}\right) = 0$. 1st A1: A second correct factor of usually $(x-4)$ or $(2x-1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark. 2nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$. 2nd A1 for correct answer of $(x+1)(2x-1)(x-4)$. **Alternative: (for the first two marks)** 1st M1: Expands $(x+1)(2x^2 + ax + b)$ {giving $2x^3 + (a+2)x^2 + (b+a)x + b$ } then compare coefficients to find values for a and b. 1st A1: $a = -9$, $b = 4$ **Not dealing with a factor of 2:** $(x+1)(x-\frac{1}{2})(x-4)$ or $(x+1)(x-\frac{1}{2})(2x-8)$ scores M1A1M1A0. **Answer only, with one sign error:** eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores M1A1M1A0. (c) **Award M1A1M1A1 for Listing all three correct factors with no working.**

Q2

Question Number	Scheme	Marks
a)i)	f(3) = $3^3 - 2 \times 3^2 - 4 \times 3 + 8$; = 5	M1; A1
ii)	f(–2) = (– 8 – 8 + 8 + 8) = 0 (B1 on Epen, but A1 in fact)	B1 (3)
	M1 is for attempt at **either** f(3) or f(–3) in (i) **or** f(–2) or f(2) in (ii).	
(b)	$[(x + 2)](x^2 - 4x + 4)$ (= 0 not required) [must be seen or used in (b)]	M1 A1
	$(x + 2)(x - 2)^2$ (= 0) (can imply previous 2 marks)	M1
	Solutions: $x = 2$ or -2 (both) or (–2, 2, 2) [**no wrong working seen**]	A1 (4) **[7]**
Notes: (a)	**No** working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate. Alternative (Long division) Divide by (x - 3) OR $(x + 2)$ to get $x^2 + ax + b$, a may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]	
(b)	First M1 requires division by a **found factor** ; e.g $(x + 2)$, $(x - 2)$ or what candidate thinks is a **factor** to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x + 2)](x^2 - 4x + 4)$ or $(x - 2)(x^2 - 4)$ Second M1:attempt to factorise **their** found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x + c)(x + d)$, *where* $\lvert cd \rvert = \lvert b \rvert$.] **N.B. Second A1 is for solutions, not factors** **SC:** (i) Answers only: Both correct, and no wrong, award M0A1M0A1 (as if B1,B1) One correct, (even if 3 different answers) award M0A1M0A0 (as if B1) (ii) Factor theorem used to find two correct factors, award M1A1, then M0, A1 if both correct solutions given. (-2,2,2 would earn all marks) (iii) If in (a) candidate has $(x+2)(x^2 - 4)$ B0, but then repeats in (b), can score M1A0M1(if goes on to factorise)A0 (answers fortuitous) *Alternative (first two marks)* $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1] $b = -4$, $c = 4$ [A1] *Method of grouping* $x^3 - 2x^2 - 4x + 8 = x^2(x-2), 4(x \pm 2)$ M1; $= x^2(x-2) - 4(x-2)$ A1 $[= (x^2 - 4)(x - 2)] = (x+2)(x-2)^2$ M1 Solutions: $x = 2$, $x = -2$ both A1	

Q3

Question Number	Scheme	Marks
(a)	$\mathrm{f}(x) = x^4 + x^3 + 2x^2 + ax + b$	
	Attempting $\mathrm{f}(1)$ or $\mathrm{f}(-1)$.	M1
	$\mathrm{f}(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$ (as required) **AG**	A1 * cso
		(2)
(b)	Attempting $\mathrm{f}(-2)$ or $\mathrm{f}(2)$.	M1
	$\mathrm{f}(-2) = \underline{16 - 8 + 8 - 2a + b = -8}$ $\{\Rightarrow -2a + b = -24\}$	A1
	Solving both equations simultaneously to get as far as $a = \ldots$ or $b = \ldots$	dM1
	Any one of $a = 9$ or $b = -6$	A1
	Both $a = 9$ and $b = -6$	A1 cso
		(5)
		[7]
	Notes	
(a)	M1 for attempting either $\mathrm{f}(1)$ or $\mathrm{f}(-1)$. A1 for applying $\mathrm{f}(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).	
(b)	M1: attempting either $\mathrm{f}(-2)$ or $\mathrm{f}(2)$. A1: correct underlined equation in a and b; eg $\underline{16 - 8 + 8 - 2a + b = -8}$ or equivalent, eg $-2a + b = -24$. dM1: an attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark. A1: any one of $a = 9$ or $b = -6$. A1: both $a = 9$ and $b = -6$ and a correct solution only.	
	Alternative Method of Long Division: (a) M1 for long division by $(x - 1)$ to give a remainder in a and b which is independent of x. A1 for $\{\text{Remainder} =\}$ $b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given.) (b) M1 for long division by $(x + 2)$ to give a remainder in a and b which is independent of x. A1 for $\{\text{Remainder} =\}$ $\underline{b - 2(a - 8) = -8}$ $\{\Rightarrow -2a + b = -24\}$. Then dM1A1A1 are applied in the same way as before.	

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions. So $n = 5 \Rightarrow n^3 + 2 = 127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\dfrac{n^3+2}{8} = \dfrac{1}{8}n^3 + \dfrac{1}{4}$ which is not a whole number
- stating $\dfrac{(n+1)^3+2}{8} = \dfrac{1}{8}n^3 + \dfrac{3}{8}n^2 + \dfrac{3}{8}n + \dfrac{3}{8}$ which is not a whole number

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so $n^3 + 2$ is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so $n^3 + 2$ cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"$n^3 + 2$ is odd and cannot be divided by 8 **exactly**"
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so $n^3 + 2$ is not divisible by 8"
"if n^3 is a multiple of 8 then $n^3 + 2$ cannot be divisible by 8"
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Q4

Question	Scheme	Marks	AOs
Algebraic approach	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=\underline{\underline{8k^3+12k^2+6k+3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3+12k^2+6k+3}}$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes
Correct expressions are required for the M's. There is no need to state "**If *n* is even,**" $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$ Some students will use $2k-1$ for odd numbers There is no requirement to change the variable. They may use $2n$ and $2n\pm1$ Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!) Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

Silver Questions

Calculators may not be used

The total mark for this section is 29

Q1

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 1 is 6 marks)

Q2

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total for Question 2 is 6 marks)

Q3

$$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(a) Write down the value of $f(k)$

(1)

When $f(x)$ is divided by $(x - 2)$ the remainder is 4

(b) Find the value of k

(2)

(c) Factorise $f(x)$ completely.

(3)

(Total for Question 3 is 6 marks)

Q4

$$f(x) = x^3 + ax^2 + bx + 3$$

where a and b are constants.

Given that when $f(x)$ is divided by $(x + 2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when $f(x)$ is divided by $(x - 1)$ the remainder is 4,

(b) find the value of a and the value of b.

(4)

(Total for Question 4 is 6 marks)

Q5

(a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \square 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b

(1)

(Total for Question 5 is 5 marks)

End of Questions

Silver Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	States or uses $f(+3) = 0$	M1	1.1b
	$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so $4x^3 - 12x^2 + 2x - 6 = (x-3)(4x^2 + ...)$	M1	2.1
	$4x^3 - 12x^2 + 2x - 6 = (x-3)(4x^2 + 2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x) So $x = 3$ is the only real root of $f(x) = 0$ *	A1*	2.4
		(4)	
			(6 marks)

Notes

(a) M1: States or uses $f(+3) = 0$

A1: See correct work evaluating and achieving zero, together with correct conclusion

(b) M1: Needs to have $(x - 3)$ and first term of quadratic correct

A1: Must be correct – may further factorise to $2(x-3)(2x^2+1)$

M1: Considers **their** quadratic for no real roots by use of completion of the square or consideration of discriminant then

A1*: a correct explanation.

Q2

Question number	Scheme	Marks
(a)	$f(-2)=2.(-2)^3-7.(-2)^2-10.(-2)+24$	M1
	$=0$ so $(x+2)$ is a factor	A1
		(2)
(b)	$\text{f}(x)=(x+2)(2x^2-11x+12)$	M1 A1
	$\text{f}(x)=(x+2)(2x-3)(x-4)$	dM1 A1
		(4)
		6 marks
Notes (a)	**M1** : Attempts $\text{f}(\pm 2)$ (Long division is M0) **A1** : is for $=0$ **and conclusion** **Note**: Stating "hence factor" or "it is a factor" or a "√" (tick) or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a preamble, eg: "If $\text{f}(-2)=0$, $(x+2)$ is a factor…." (Not just f(-2)=0)	
(b)	**1st M1**: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection." Or *Alternative Method* : **1st M1**: Use $(x+2)(ax^2+bx+c)=2x^3-7x^2-10x+24$ with expansion and comparison of coefficients to obtain $a=2$ and to obtain values for b and c **1st A1**: For seeing $(2x^2-11x+12)$. [Can be seen here in (b) after work done in (a)] **2nd M1**: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and **needs factors** **2nd A1**: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+2)\}(2x^2-11x+12)$ to $\{x=-2\}, x=\frac{3}{2}, 4$, and not list all three factors. Award these responses M1A1M0A0. Finds $x=4$ and $x=1.5$ by factor theorem, formula or calculator and produces factors M1 $\text{f}(x)=(x+2)(2x-3)(x-4)$ **or** $\text{f}(x)=2(x+2)(x-1.5)(x-4)$ **o.e. is full marks** $\text{f}(x)=(x+2)(x-1.5)(x-4)$ **loses last A1**	

Pearson

Q3

Question Number		Scheme	Marks	
Q	(a)	$\mathrm{f}(k) = -8$	B1	(1)
	(b)	$\mathrm{f}(2) = 4 \Rightarrow \quad 4 = (6-2)(2-k) - 8$	M1	
		So $\quad k = -1$	A1	(2)
	(c)	$\mathrm{f}(x) = 3x^2 - (2+3k)x + (2k-8) \quad = 3x^2 + x - 10$	M1	
		$= (3x - 5)(x + 2)$	M1A1	(3)
				[6]

(b) M1 for substituting $x = 2$ (not $x = -2$) and equating to 4 to form an equation in k.
If the expression is expanded in this part, condone 'slips' for this M mark.
Treat the omission of the -8 here as a 'slip' and allow the M mark.

Beware:
Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$.

Alternative:
M1 for dividing by $(x-2)$, to get $3x +$ (function of k), with remainder as a function of k, and equating the remainder to 4. [Should be $3x + (4-3k)$, remainder $-4k$].

No working:
$k = -1$ with no working scores M0 A0.

(c)

1st M1 for multiplying out and substituting their (constant) value of k (in either order).
The multiplying-out may occur earlier.
Condone, for example, sign slips, but if the 4 (from part (b)) is included in the f(x) expression, this is M0. The 2nd M1 is still available.

2nd M1 for an attempt to factorise their three term quadratic (3TQ).

A1 The correct answer, as a product of factors, is required.
Allow $3\left(x - \frac{5}{3}\right)(x+2)$

Ignore following work (such as a solution to a quadratic equation).
If the 'equation' is solved but factors are never seen, the 2nd M is not scored.

Q4

Question number	Scheme	Marks
(a)	$\mathrm{f}(-2) = -8+4a-2b+3=7$ so $2a-b=6$ *	M1 A1 (2)
(b)	$\mathrm{f}(1)=1+a+b+3=4$ Solve two linear equations to give $a=2$ and b = -2	M1 A1 M1 A1 (4) **6**
Notes	(a) **M1** : Attempts f(± 2) = 7 or attempts long division as far as putting remainder equal to 7 (There may be sign slips) **A1** is for correct equation with remainder = 7 and for the printed answer with no errors and no wrong working between the two (b) **M1** : Attempts f(± 1) = 4 or attempts long division as far as putting remainder equal to 4 **A1** is for correct equation with remainder = 4 and powers calculated correctly **M1** : Solving simultaneous equations (may be implied by correct answers). This mark may be awarded for attempts at elimination or substitution leading to values for both a and b. Errors are penalised in the accuracy mark. **A1** is cao for values of a and b and explicit values are needed. Special case: Misreads and puts remainder as 7 again in (b). This may earn M1A0M1A0 in part (b) and will result in a maximum mark of 4/6	
Long Divisions	$\begin{array}{r\|l} & x^2+(a-2)x+(b-2a+4) \\ \hline (x+2) & x^3+ax^2+bx+3 \\ & x^3+2x^2 \\ & \ldots\ldots \end{array}$ and reach their "3 - $2b$ + $4a$ - 8" = 7 M1 $\begin{array}{r\|l} & x^2+(a+1)x+(b+a+1) \\ \hline (x-1) & x^3+ax^2+bx+3 \\ & x^3-x^2 \\ & \ldots\ldots \end{array}$ and reach their "3 + b + a + 1" = 4 M1 A marks as before	

Q5

Question	Scheme	Marks	AOs
(a)	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2+b^2 \geqslant 4ab$	A1	1.1b
	(As $a>0, b>0$) $\dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b}+\dfrac{b}{a} \geqslant 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \dfrac{4a}{b}+\dfrac{b}{a}=-20-\dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
		(5 marks)	

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2+b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Gold Questions

Calculators may not be used

The total mark for this section is 27

Q1

$$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total for Question 1 is 6 marks)

Q2

$$f(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when $f(x)$ is divided by $(x - 2)$ is equal to the remainder when $f(x)$ is divided by $(x + 1)$.

(a) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of b.

(3)

(Total for Question 2 is 8 marks)

Q3

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

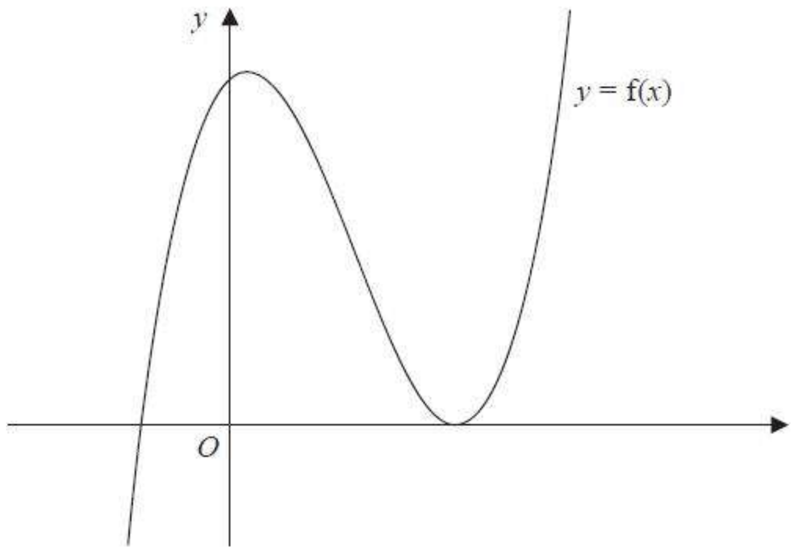

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

(Total for Question 3 is 10 marks)

Q4

(a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

(Total for Question 4 is 3 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	Attempt $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ Use of long division is M0A0 as factor theorem was required. $\mathrm{f}(-3) = 162 - 63 - 120 + 21 = 0$ so $(x+3)$ is a factor	M1 A1 **(2)**
(b)	Either (Way 1): $\mathrm{f}(x) = (x+3)(-6x^2 + 11x + 7)$ $= (x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x = -1/2$ **or** $x = 7/3$ Uses trial or factor theorem to obtain both $x = -1/2$ **and** $x = 7/3$ Puts three factors together (see notes below) Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working	M1A1M1A1 **(4)**

	Notes
(a)	M1 for attempting either $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ – with **numbers substituted into expression** A1 for calculating $\mathrm{f}(-3)$ **correctly to 0**, and they must state $(x+3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if $\mathrm{f}(-3) = 0$, $(x+3)$ is a factor". $-6(-3)^3 - 7(-3)^2 + 40(-3) + 21 = 0$ so $(x+3)$ is a factor of $\mathrm{f}(x)$ is M1A1 providing bracketing is correct.
(b)	1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b). 1st A1: usually for $(-6x^2 + 11x + 7)\ldots$ Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$ but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised. Ignore subsequent work (such as a solution to a quadratic equation.) Way 2: The second M mark needs three roots together so $\pm 6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained α and β by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0. N.B. Replacing $(-6x^2 + 11x + 7)$ (already awarded M1A1) by $(6x^2 - 11x - 7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0

Q2

Question Number	Scheme	Marks
(a)	$\mathrm{f}(2) = 16 + 40 + 2a + b$ **or** $\mathrm{f}(-1) = 1 - 5 - a + b$	M1 A1
	Finds 2nd remainder and equates to 1st $\Rightarrow$ $16 + 40 + 2a + b = 1 - 5 - a + b$	M1 A1
	$a = -20$	A1cso (5)
(b)	$\mathrm{f}(-3) = (-3)^4 + 5(-3)^3 - 3a + b = 0$	M1 A1ft
	$81 - 135 + 60 + b = 0$ gives $b = -6$	A1 cso
		(3) **[8]**
Alternative for (a)	(a) Uses long division, to get remainders as $b + 2a + 56$ or $b - a - 4$ or correct equivalent	M1 A1
	Uses second long division as far as remainder term, to get $b + 2a + 56 = b - a - 4$ or correct equivalent	M1 A1
	$a = -20$	A1cso (5)
Alternative for (b)	(b) Uses long division of $x^4 + 5x^3 - 20x + b$ by $(x + 3)$ to obtain $x^3 + 2x^2 - 6x + a + 18$ (with their value for a)	M1 A1ft
	Giving remainder $b + 6 = 0$ and so $b = -6$	A1 cso
		(3) **[8]**
Notes (a)	**M1** : Attempts $\mathrm{f}(\pm 2)$ or $\mathrm{f}(\pm 1)$ **A1** is for the answer shown (or simplified with terms collected) for one remainder **M1:** Attempts other remainder and puts one equal to the other **A1:** for correct equation in a (and b) then **A1 for** $a = -20$ **cso**	
(b)	**M1** : Puts $\mathrm{f}(\pm 3) = 0$ **A1** is for f(-3) = 0, (where f is original function), with no sign or substitution errors (follow through on 'a' and could still be in terms of a) **A1:** b = -6 is cso.	
Alternatives	(a) **M1**: Uses long division of $x^4 + 5x^3 + ax + b$ by $(x \pm 2)$ **or** by $(x \pm 1)$ as far as three term quotient **A1**: Obtains at least one correct remainder **M1:** Obtains second remainder and puts two remainders (no x terms) equal **A1**: correct equation **A1**: correct answer a = -20 following correct work. (b) **M1**: complete long division as far as constant (ignore remainder) **A1ft:** needs correct answer for their a **A1**: correct answer	

Beware: It is possible to get **correct answers with wrong working**. If remainders are equated to 0 in part (a) both correct answers are obtained fortuitously. This could score M1A1M0A0A0M1A1A0

Q3

Question	Scheme	Marks	AOs
(a)	Attempts $f(4)=2\times4^3-13\times4^2+8\times4+48$	M1	1.1b
	$f(4)=0\Rightarrow(x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3-13x^2+8x+48=(x-4)\left(2x^2...x-12\right)$	M1	2.1
	$=(x-4)\left(2x^2-5x-12\right)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$f(x)=(x-4)^2(2x+3)\Rightarrow f(x)=0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1	2.2a
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k=\pm4,\pm\frac{3}{2}$	M1	1.1b
	$k=4,-\frac{3}{2}$	A1ft	1.1b
		(2)	
			(10 marks)

Notes
(a) **M1:** Attempts to calculate $\mathrm{f}(4)$. Do not accept $\mathrm{f}(4)=0$ without sight of embedded values or calculations. If values are not embedded look for two correct terms from $\mathrm{f}(4)=128-208+32+48$ Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection. See below for awarding these marks. **A1:** Correct reason with conclusion. Accept $\mathrm{f}(4)=0$, hence factor as long as M1 has been scored. This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $\mathrm{f}(4)=0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe. If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor **(b)** **M1:** Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

So for inspection award for $2x^3-13x^2+8x+48=(x-4)(2x^2...x\pm12)$ For division look for $\begin{array}{r} 2x^2-5x \\ x-4\overline{)2x^3-13x^2+8x+48} \\ \underline{2x^3-8x^2} \\ -5x^2 \end{array}$ **A1:** Correct quadratic factor $(2x^2-5x-12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark. **If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2-5x-12)$.** **dM1:** Correct attempt to solve or factorise their $(2x^2-5x-12)$ including use of formula Apply the usual rules $(2x^2-5x-12)=(ax+b)(cx+d)$ where $ac=\pm2$ and $bd=\pm12$ Allow the candidate to move from $(x-4)(2x^2-5x-12)$ to $(x-4)^2(2x+3)$ for this mark.

dM1: Correct attempt to solve or factorise their $(2x^2-5x-12)$ including use of formula

Apply the usual rules $(2x^2-5x-12)=(ax+b)(cx+d)$ where $ac=\pm2$ and $bd=\pm12$

Allow the candidate to move from $(x-4)(2x^2-5x-12)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $\text{f}(x)=(x-4)(2x+3)(x-4)$ or $\text{f}(x)=(x-4)^2(2x+3)$ or

$\text{f}(x)=2(x-4)^2\left(x+\dfrac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x=4$ and $-\dfrac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $\text{f}(x)=(x-4)^2(2x+3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorsises to $(x-4)(2x^2-5x-12)$ and solves $2x^2-5x-12=0 \Rightarrow x=4,-\dfrac{3}{2}$ followed by an explanation that the roots are $4,4,-\dfrac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction.**

Accept **either** there are 3 roots **or** state that it is a solution of $\text{f}(x)=2$ or $\text{f}(x)-2=0$

A1: Fully explains:

Eg. States three roots, as $\text{f}(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $\text{f}(x)=2$ (You may see $y=2$ drawn on the diagram)

(d)

M1: For sight of ±4 **and** $\pm\dfrac{3}{2}$ Follow through on $\pm$ their roots.

A1ft: $k=4,-\dfrac{3}{2}$ Follow through on their roots. Accept $4,-\dfrac{3}{2}$ but not $x=4,-\dfrac{3}{2}$

Q4

Question	Scheme	Marks	AOs
(a)	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2+b^2 \geqslant 4ab$	A1	1.1b
	(As $a>0, b>0$) $\dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b}+\dfrac{b}{a} \geqslant 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \dfrac{4a}{b}+\dfrac{b}{a}=-20-\dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2+b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Platinum Questions

Calculators may not be used

The total mark for this section is 9

1 (*a*) Show that $(x + 1)$ is a factor of $2x^3 + 3x^2 - 1$

(1)

(*b*) Solve the equation

$$\sqrt{x^2+2x+5} = x + \sqrt{2x+3}$$

(8)

(Total for Question 1 is 9 marks)

End of Questions

Platinum Mark Scheme

1

(a)	$-2+3-1=0$ so $(x+1)$ is a factor	B1cso (1)	
(b)	$x^2+2x+5=\underline{x^2}+\underline{2x\sqrt{2x+3}}+\underline{2x+3}$	M1	Attempt to square. 3 terms on RHS
	$1=x\sqrt{2x+3}$	M1	Prepare for final sq
	$0=2x^3+3x^2-1$ (Accept $2x^3+3x^2=1$ o.e.)	A1cso	
	$0=(x+1)(2x^2+x-1)$	M1	Div attempt. At least 2 correct terms of quadratic
	$0=(x+1)(2x-1)(x+1)$		
	$\underline{x}=-1$ or $\frac{1}{2}$	A1	Correct factors and both roots
	Check -1: LHS $=2$ RHS $=0$ **so -1 is not a solution**	B1	Must reject -1
	Check $\frac{1}{2}$: LHS $=\sqrt{\frac{25}{4}}=\frac{5}{2}$ RHS$=\frac{1}{2}+\sqrt{4}=2.5$	M1	Attempts 0.5 in original or line 2
	(Only) solution is 0.5	A1 (8)	Only award if check is in <u>original</u>
	[S- for treating $\sqrt{4}$ as ± 2 etc]	**[9]**	